UDK 669+621.7

ANISOTROPY OF COMPUTATIONALLY TAILORED MICROSTRUCTURES WITH NEGATIVE POISSON'S RATIO

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Introduction

Materials that expand laterally under tension and/or contract when under compression are known as materials with negative Poisson's ratio or auxetic materials. Negative Poisson's ratios were first reported for single crystalline iron pyrite in 1944 (Love, 1944). At that time it was believed that negative values of Poisson's ratio could not be achieved and this phenomenon was ascribed to the twining defects in the pyrite crystals. In 1987, Lakes discovered the negative Poisson's ratio polyurethane (PU) foam with re-entrant structure (Lake, 1987). A limited number of auxetic materials exist naturally. An example of natural auxetic material is bone. The negative Poisson's ratio of bone was measured by Williams and Lewis (1982). Auxetic materials have many potential applications in several practical fields such as automotive and aerospace industries, smart mattresses and in manufacturing smart filters. They have also shown great potential for applications in sensors, molecular sieves and as structural materials (Yang et al., 2004). Since the discovery of auxetic materials, investigations have been focused on examining their properties and applications, finding new materials with negative Poisson's ratio and developing new procedures to produce such materials. A number of methods have been proposed to find new auxetic materials, some of which rely on transforming non-auxetic materials into an auxetic form (foams), whereas others employ structural techniques and new architectural shapes to achieve the auxetic effect by changing the shape of the base cell of the material. Sigmund (1995) presented a method for tailoring 2D and 3D materials with prescribed elastic properties including negative Poisson's ratio. The problem was formulated as an optimisation problem of minimising the structural weight of the base cell with a prescribed constitutive property (e.g. negative Poisson's ratio). It was shown the proposed numerical method can be used to tailor materials and find the microstructures with desired properties. Matsuoka et al. (2001) presented a method based on a genetic algorithm and finite element analysis to design structures of composites and mixtures with prescribed elastic properties. They used a 2D plane stress finite element model to analyse a mixture structure with two different materials. In the model each cell was assigned to one of the two materials. The objective function was defined as minimising the difference between Poisson's ratios in X and Y directions and a predefined Poisson's ratio of -0.3.

In this paper a methodology is presented to design microstructures of auxetic materials with a wide range of different Poisson's ratios. The proposed methodology is based on a combination of finite element method and an evolutionary optimisation algorithm. In addition the anisotropy level and fluctuation of material properties of these microstructures will be examined in order to evaluate the possible relationship between negative Poisson's ratio and anisotropy. It will be shown that using the developed methodology it is possible to maintain the isotropy of these microstructures while they still exhibit auxetic behaviour.

2. Evolutionary optimisation of auxetic microstructures

The developed methodology to design auxetic microstructures in this study is based on a combination of the finite element method and a genetic algorithm. In general optimisation problem involves minimisation (or maximisation) of an objective function subject to a set of constraints. Here the problem is formulated as an optimisation problem of finding microstructures with prescribed behavioural requirements (in this case negative Poisson's ratio). Different microstructures are generated and evolved using the principles of genetic algorithm and the behaviour of each structure is analysed using the finite element method to evaluate its fitness in competition with other generated structures. During successive generations, the genetic algorithm operators gradually improve the fitness of the solution and direct the search towards the optimal or near optimal solution.

An 11 by 11 regular initial structure as shown in Figure 1 is considered as starting grid for the GA. Due to symmetry only a quarter of structure is modelled and therefore appropriate boundary conditions are provided on left and bottom boundaries of the model. The model is made of 274 nodes and 399 elements. The optimisation variables were considered as the X and Y coordinates of the nodes and minimising Poisson's ratio, ν_{12}, was the objective function. A number of uniform longitudinal point loads were applied through the top edge of the structure in the FE model and the average values of the Poisson's ratio (ν_{12}) and Young modulus (E_1) were calculated from the simulated average longitudinal and lateral displacements of the top and side nodes of the microstructure. This was also repeated for the lateral direction to calculate ν_{21} and E_2. In addition two shear tests were simulated to measure shear modulus (G_12, G_21) of the microstructure. Since the nodes are allowed to move in both 1 and 2 directions, the number of variables is 460. The population size and number of generations were set to 50 and 500 respectively which were found to be adequate after a number of trial and error runs. After the optimisation terminated, the minimum achieved Poisson's ratio (ν_{12}) was -1.162. The evolved structure and its deformed shape are illustrated in Figures 2a and b respectively.

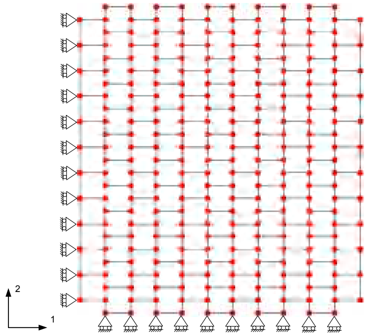


Fig. 1. 11 by 11 cell representing a symmetric quarter of the initial structure

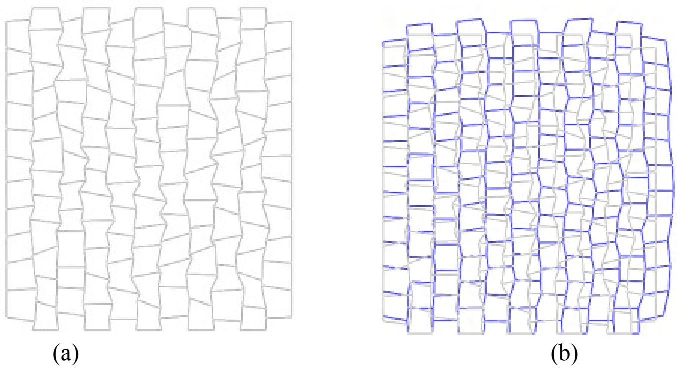


Fig. 2. (a) Evolved microstructure (b) evolved microstructure together with its deformed shape superimposed

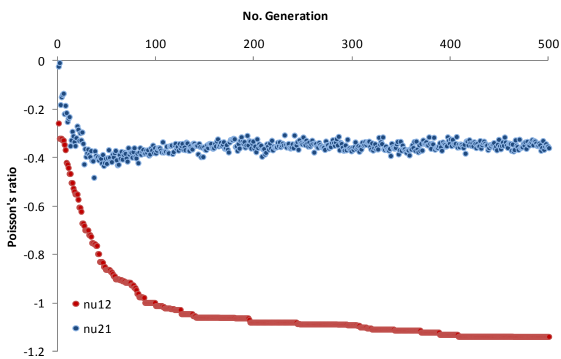


Fig. 3. variation of Poisson's ratios (ν_{12} and ν_{21}) during the genetic algorithm generations

In each generation, the best solution within the population of solutions was recorded which is shown in Figure 3 (red points). In this figure the fluctuation of ν_{12} and ν_{21} versus the number of generation is depicted. In this figure it can be seen that ν_{12} is continuously decreasing since it was the objective function for the GA while ν_{21} is varying randomly.

3. Anisotropy of auxetic materials

Both isotropic and anisotropic behaviour for materials with negative Poisson's ratio has been reported by many researchers (Lakes, 1993). Lake (1987) fabricated isotropic foam structure with negative Poisson's ratio. Caddock and Evans (1989) reported anisotropic microcellular foams that exhibit negative Poisson's ratio. Alderson and Evans (1992) produced a microporous polyethylene with large negative Poisson's ratio. The developed polyethylene was anisotropic with Poisson's ratio of -1.24 for compression in the radial direction. Masters and Evans (1996) developed theoretical models to predict the elastic constants of honeycomb cells and discussed the level of anisotropy for each model. It appears that there is no certain relationship between the level of anisotropy and auxetic behaviour in general and it is possible to develop materials or microstructures with negative Poisson's ratio having both isotropic and anisotropic behaviour.

For the auxetic microstructures developed in this paper, the level of anisotropy was measured by monitoring the fluctuation of Poisson's ratio and elastic modulus of best solutions in each generation in both directions 1 and 2. In addition G_{12} and G_{21} of the best solutions among populations at each generation were also recorded. The variations of the $\frac{\nu_{12}}{\nu_{21}}$, $\frac{E_{11}}{E_{22}}$ and $\frac{G_{12}}{G_{21}}$ with number of generations are shown in Figure 4.

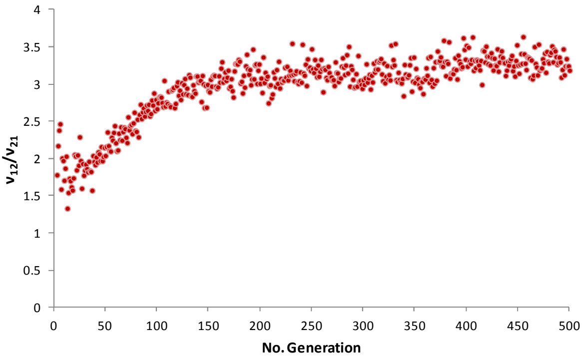
Figures 4 (a) and (b) show that as the number of generations increases, the ratios $\left(\frac{\nu_{12}}{\nu_{21}}\right)$ and $\left(\frac{E_{11}}{E_{22}}\right)$ deviate from 1 which shows that the degree of anisotropy increases as the value of Poisson's ratio decreases. The ratio G_{12} / G_{21} fluctuates randomly between 0.5 and 1.

4. Imposing Isotropy

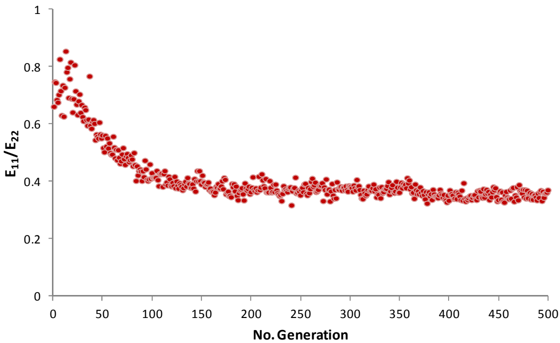
Usually materials with negative Poisson's ratio have regular microstructures although there are some exceptions. Generally the disorder in a material is seen as something uninvited, because it is believed that disorder or heterogeneity will tend to reduce the range of material properties available (Gasper et al. 2009). In order to enforce isotropic behaviour in directions 1 and 2 during the optimisation of microstructure of auxetic materials, the objective function in the GA-FEM process was changed to the following:

$$Minimise\left(\nu_{12}+\alpha_1\left|\frac{\nu_{12}}{\nu_{21}}-1\right|+\alpha_2\left|\frac{E_{11}}{E_{22}}-1\right|+\alpha_3\left|\frac{G_{12}}{G_{21}}-1\right|\right) \quad (1)$$

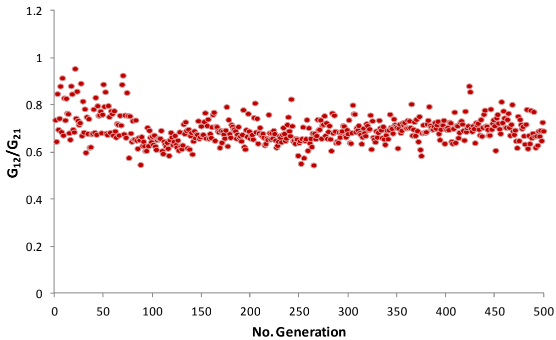
where α_1, α_2 and α_3 are values greater than 1 (penalties) that force the GA to keep the values of ν_{12} and ν_{21} as well as E_{11} and E_{22} and G_{12} and G_{21} close to each other while ν_{12} is decreasing.



(a)



(b)



(c)

Fig. 4. (a) $\frac{\nu_{12}}{\nu_{21}}$ vs. No. Generation (b) $\frac{E_{11}}{E_{22}}$ vs. No. Generation (c) $\frac{G_{12}}{G_{21}}$ vs. No. Generation

This should result in optimisation of microstructures with auxetic behaviour with similar material behaviour in 1 and 2 directions. The above objective function was used to optimise the microstructure shown in Figure 1 with $\alpha_1 = \alpha_2 = \alpha_3 = 20$. A fter 500 generations the final value achieved for Poisson's ratio was -0.32 (note that this was not necessarily the minimum Poisson's ratio since the objective function was different (equation 1). The variations of Poisson's ratio and other material parameters with number of generations are shown in Figure 5.

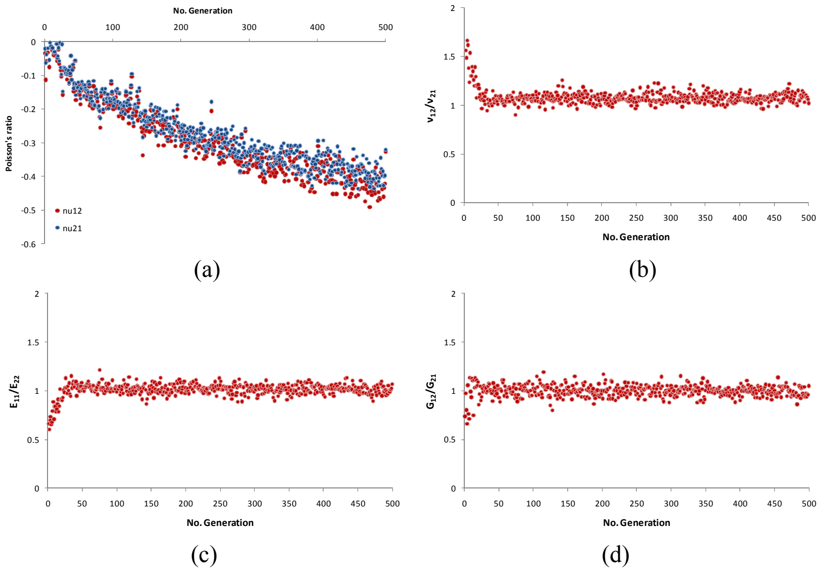


Fig. 5. Variations of Poisson's ratio, elastic modulus and shear modulus against number of generations

Figure 5a shows the variation of Poisson's ratios (ν_{12} and ν_{12}) with number of generations. Although these values are fluctuating in a random way but the general trend shows that microstructures with lower values of Poisson's ratio are evolved by increasing the number of generations. In Figures 5b, 5c and 5d it can be seen that after almost 50 generations the values of $\frac{\nu_{12}}{\nu_{21}}$, $\frac{E_{11}}{E_{22}}$, and $\frac{G_{12}}{G_{21}}$ are moving toward the value of 1 which is an indication that the evolved microstructures are reaching a similar behaviour in 1 and 2 directions. Although it was possible to choose greater values for α_1, α_2 and α_3 in order to obtain more isotropy, however this can result in microstructures with less auxetic behaviour and therefore a compromise should be made in the selection of α_i values.

5. Summary and conclusion

Auxetic materials differ from conventional materials by the manner in which they respond to stretching; they tend to get fatter when stretched, resulting in a negative Poisson's ratio. These materials with their improved properties have proved their efficiency in several practical applications. In this paper a numerical methodology was presented for design of microstructure of auxetic materials with a wide range of different negative Poisson's ratios. The proposed methodology is based on the combination of finite element method and a genetic algorithm. In this approach, different microstructures are generated and evolved using a genetic algorithm and the behaviour of each microstructure is analysed using the finite element method to evaluate its fitness in competition with other generated structures.

One of the advantages of this method is that it gives choice from a set of feasible solutions, rather than a single solution, in design of auxetic materials. The methodology gives a wide range of different microstructures with similar Poisson's ratios from which the most appropriate one can be selected by the user based on engineering judgment. Another advantage is that no initial guess is required for the microstructure to start the optimisation and optimal structures can be evolved from any arbitrary initial structure. Moreover at the end of the process the results of optimisation can be directly used and no additional effort is required for extraction or modification of the identified structures.

The fluctuation of microstructure properties were monitored during the evolutionary optimisation process and it was observed that the degree of anisotropy generally increases as Poisson's ratio decreases. However it was shown that by defining a suitable objective function, the problem can be formulated of finding auxetic microstructures with isotropic behaviour.

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