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METHODOLOGY OF THE PROCESS DESIGN OF GROSS PRODUCT OUTPUT OF AN ENTERPRISE

SHIBKO O. ^{1*}, *PhD, Associate Prof.*ERSHOVA N. ^{2*}, *DR. SC. (TECH.), PROF.*VELMAGINA N. ^{3*}, *Ph. D., Associate Prof.*

^{1*} Department of Applied Mathematics and Information Technologies. Pridniprovsk State Academy of Civil Engineering and Architecture. 24-a Chernishevskogo st. 49600, Dnipropetrovsk, Ukraine, tel. +38 (0567) 56-34-10, email: prmat@mail.pgasa.dp.ua
ORCID ID: 0000-0001-5894-0642

^{2*} Department of Applied Mathematics and Information Technologies. Pridniprovsk State Academy of Civil Engineering and Architecture. 24-a Chernishevskogo st. 49600, Dnipropetrovsk, Ukraine, tel. +38 (0567) 56-34-10, email: nersova107@gmail.com,
ORCID ID: 0000-0003-0198-0883

^{3*} Department of Applied Mathematics and Information Technologies. Pridniprovsk State Academy of Civil Engineering and Architecture. 24-a Chernishevskogo st. 49600, Dnipropetrovsk, Ukraine, tel. +38 (0567) 56-34-10, email: velmagina24@gmail.com
ORCID ID: 0000-0002-5584-3748

Annotation. The methods for designing the process of gross product output of an enterprise, based on the stochastic method of dynamic programming for continuous deterministic systems was created. The selection of weight factors of quadratic quality functional was substantiated, the dependence between parameters of design and weight factors was established, and the region of optimum values of design parameters was determined, the optimal filter of the process of gross product output of an enterprise was obtained. Modeling of the process of gross product output with optimum values of coefficients of growth and retirement of basic production assets. Production capacity is determined by the output at the achieved level of organization and production technology. The shape of production capacity graphs changes noticeably even with minor changes in parameters, i.e., capacity is sensitive to changes in the state of a firm. Design parameters are determined by the matrix method of dynamic programming. Based on the concepts of observability and controllability, it has been established that the process of producing a gross product of an enterprise is subject to management. By modeling and calculations, it was proved that using the matrix method of dynamic programming, one can obtain analytical dependencies for growth and retirement coefficients, as well as calculate the regions of their optimal values, i.e. The production capacity of an enterprise can be the main indicator of the characteristics of the life cycle of an enterprise. The technique should be used when designing the process of producing a gross product of enterprises, its implementation will reduce the design time and ensure the stability of the process of producing a gross product.

Keywords: gross product, dynamic programming, quadratic functional of quality, weight factors, design parameters, optimal filter

РОЗРОБКА МЕТОДИКИ ПРОЕКТУВАННЯ ПРОЦЕСУ ВИПУСКУ ВАЛОВОГО ПРОДУКТУ ПІДПРИЄМСТВА

ШИБКО О.М. ^{1*}, *к.т.н., доцент*ЕРШОВА Н. М. ^{2*}, *д. т. н., проф*ВЕЛЬМАГІНА Н. О. ^{3*}, *к.ф-м.н., доцент*

^{1*} Кафедра «Прикладна математика та інформаційні технології». Державний вищий навчальний заклад «Придніпровська державна академія будівництва та архітектури», вул. Чернишевського, 24-а, 49600, Дніпро, Україна, тел. +38 (0567) 56-34-10, email: prmat@mail.pgasa.dp.ua ORCID ID: 0000-0001-5894-0642

^{2*} Кафедра «Прикладна математика та інформаційні технології». Державний вищий навчальний заклад «Придніпровська державна академія будівництва та архітектури», вул. Чернишевського, 24-а, 49600, Дніпро, Україна, тел. +38 (0567) 56-34-10, email: nersova107@gmail.com, ORCID ID: 0000-0003-0198-0883

^{3*} Кафедра «Прикладна математика та інформаційні технології». Державний вищий навчальний заклад «Придніпровська державна академія будівництва та архітектури», вул. Чернишевського, 24-а, 49600, Дніпро, Україна, тел. +38 (0567) 56-34-10, email: velmagina24@gmail.com ORCID ID: 0000-0002-5584-3748

Анотація. Створено методику проектування процесу випуску валової продукції підприємства на основі стохастичного методу динамічного програмування для неперервних детерміністичних систем. Обґрунтовано вибір вагових факторів функціональності квадратичного якості, встановлена. Залежність між параметрами конструкції та ваговими коефіцієнтами, визначено область оптимальних значень проектних параметрів, оптимальний фільтр процесу валової продукції підприємства був отриманий. Моделювання процесу випуску валової продукції з оптимальними значеннями коефіцієнтів зростання та виходу на пенсію основних виробничих фондів. Виробнича потужність визначається по випуску продукції на досягнутому рівні організації і технології виробництва. Форма графіків виробничої потужності помітно змінюється навіть при незначних змінах параметрів, т. е. Потужність чутлива до зміни стану фірми. Параметри проектування визначаються матричним методом динамічного програмування. На основі понять спостережливості і керованості встановлено, що процес випуску валового продукту підприємства є об'єктом управління. Шляхом моделювання та розрахунків доведено, що за

допомогою матричного методу динамічного програмування можна отримати аналітичні залежності для коефіцієнтів росту і вибуття, а також розрахувати області їх оптимальних значень, тобто виробнича потужність підприємства може бути основним показником характеристики життєвого циклу підприємства. Методику доцільно використовувати при проектуванні процесу випуску валового продукту підприємств, її впровадження скоротить час проектування і забезпечить стабільність процесу випуску валового продукту.

Ключові слова: валовий продукт, динамічне програмування, квадратичний функціонал якості, вагові фактори, проектні параметри, оптимальний фільтр.

РАЗРАБОТКА МЕТОДИКИ ПРОЕКТИРОВАНИЯ ПРОЦЕССА ВЫПУСКА ВАЛОВОГО ПРОДУКТА ПРЕДПРИЯТИЯ

ШИБКО О.Н. ^{1*}, *к.т.н., доцент*
ЕРШОВА Н. М. ^{2*}, *д. т. н., проф.*
ВЕЛЬМАГИНА Н. А. ^{3*}, *к.ф-м.н., доцент*

^{1*} Кафедра «Прикладная математика и информационные технологии». Государственное высшее учебное заведение «Приднепровская государственная академия строительства и архитектуры», ул. Чернышевского, 24-а, 49600, Днепр, Украина, тел. +38 (0567) 56-34-10, email: prmat@mail.pgasa.dp.ua ORCID ID: 0000-0001-5894-0642

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Аннотация. Создана методика проектирования процесса выпуска валовой продукции предприятия на основе стохастического метода динамического программирования для непрерывных детерминированных систем. Обоснован выбор весовых коэффициентов квадратичного функционала качества, установлена зависимость между параметрами расчетного и весового коэффициентов, определена область оптимальных значений расчетных параметров, оптимальным фильтром процесса валовой продукции предприятия является получен. Моделирование процесса производства валовой продукции с оптимальными значениями коэффициентов роста и выхода на основные производственные активы. Производственная мощность определяется по выпуску продукции на достигнутом уровне организации и технологии производства. Форма графиков производственной мощности заметно изменяется даже при незначительных изменениях параметров, т. е. мощность чувствительна к изменению состояния фирмы. Параметры проектирования определяются матричным методом динамического программирования. На основе понятий наблюдаемости и управляемости установлено, что процесс выпуска валовой продукта предприятия является объектом управления. Путем моделирования и расчетов доказано, что с помощью матричного метода динамического программирования можно получить аналитические зависимости для коэффициентов роста и выбытия, а также рассчитать области их оптимальных значений, т.е. производственная мощность предприятия может быть основным показателем характеристики жизненного цикла предприятия. Методику целесообразно использовать при проектировании процесса выпуска валового продукта предприятий, ее внедрение сократит время проектирования и обеспечит стабильность процесса выпуска валового продукта.

Ключевые слова: валовой продукт, динамическое программирование, квадратичный функционал качества, весовые коэффициенты, проектные параметры, оптимальный фильтр

Introduction

The process of gross product manufacturing is the basis of any enterprise. It must be continuous in time, i. e. it can be described by an ordinary differential equation. Production of gross product is impossible without investment of the basic production assets (BPA). Therefore, parameters of the process of manufacturing of gross product must include coefficients of growth and retirement of BPA. This model is constructed in [2]. The problem arises: how to determine the values of these parameters so that the process of gross product manufacturing could be sustainable. This problem has not been solved, although it is rather urgent.

This paper proposes the procedure for designing the process of gross product output of an enterprise, based on

stochastic method of dynamic programming for continuous deterministic systems.

Research goal and objectives

The goal of the present research is to create a technology of design of the process of gross product manufacturing by an enterprise, the parameters of which are determined using the method of dynamic programming for continuous stochastic systems.

To accomplish the set goal, the following tasks were to be solved:

- to prove that the process of gross product manufacturing by the company is a control object;
- to determine analytical dependences for design parameters using the matrix method of dynamic programming;

- to substantiate the choice of weight factors of the quadratic functional of quality;
- to determine the region of optimal values of design parameters;
- to obtain the optimal filter of the process of gross product manufacturing;
- to perform mathematical modeling of the process of gross product manufacturing with optimum values of coefficients of BPA growth and retirement.

Development of mathematical models and methods of design

1. Determining of analytical dependences for design parameters with the help of the matrix method of dynamic programming

The result of production activity of an enterprise is the gross product. Mathematical model of the process of manufacturing of gross product is [2]:

$$\frac{d^2Y}{dt^2} + (\alpha - s) \frac{dY}{dt} + cY = \frac{\mu}{qm} V, \quad Y(t_0) = Y_0; \quad (1)$$

$$\frac{dm}{dt} = -sm, \quad m(t_0) = m_0.$$

Here m is the capital ratio of BPA for manufacturing of given products; $\beta = \alpha m$ is the coefficient of BPA retirement; s is the generalized technical and economic indicator of reflection of the level of scientific and technical development of an enterprise; c is the coefficient of BPA growth; Y is the gross product; $V(t)$ is the instantaneous value of basic assets; μ , q are the coefficients of depreciation and proportionality (parameter of the model). Model (1) can be used to analyze the process of gross product manufacturing at different values of its parameters. The model includes a large number of parameters. There arises the problem how in the process of design to determine a range of a change in parameters, in which the process of manufacturing gross product will be sustainable. Sustainability is an internal property of a process independent of instantaneous values of basic assets. An enterprise and a process are only created, so it is not possible to judge about their level of scientific and technological development. As an assumption, we will take capital ratio as constant. Then the mathematical model of the process of gross product manufacturing will take the form:

$$m\ddot{Y} + \beta\dot{Y} + cY = 0 \quad (2)$$

This model can be accepted as the model-analogue in optimization of parameters by the methods of the optimal control theory [8].

We set the goal to define calculation formulas for design parameters: coefficients of BPA retirement and growth at constant capital ratio.

In the matrix method of dynamic programming the differential equation of the designed process is written down as

$$m\ddot{Y} = -F \quad \text{or} \quad \ddot{Y} = -F/m = -u, \quad (3)$$

where u is the unknown synthesizing function, representing acceleration of the process of gross product manufacturing. Let us introduce designations: $x_1 = Y$; $x_2 = \dot{Y}$, then we have

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= -u. \end{aligned} \quad (4)$$

Let us write down system (4) in the matrix form

$$\dot{X} = AX + BU, \quad (5)$$

where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} Y \\ \dot{Y} \end{bmatrix}$; $U = [u]$; $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

As the optimality criterion, we will accept quadratic functional of quality:

$J = \int_0^{\infty} V dt = \int_0^{\infty} (\alpha x_1^2 + \gamma x_2^2 + \mu u^2) dt$ or in the matrix form

$$J = \int_0^{\infty} V dt = \int_0^{\infty} (X'PX + U'GU) dt, \quad (6)$$

where α, γ, μ are the weight factors of the functional. Matrices P, G of functional (6) have the form:

$$P = \begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix}; \quad G = [\mu].$$

The problem is stated: to determine synthesizing function of equation (3), which will minimize the functional of quality. The physical sense of the functional is monetary costs to maintain stability of the process of gross product manufacturing [4].

A necessary optimality condition is solution of the nonlinear algebraic Riccati equation

$$P + A' \cdot S + SA - SBG^{-1}B'S = 0. \quad (7)$$

We will write down equation (7) in the expanded form

$$\begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \left[\frac{1}{\mu} \right] \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} = 0.$$

Hence we will obtain the system of non-linear equations for determining the elements of the symmetric matrix S .

$$\begin{cases} \alpha - S_{12}^2 / \mu = 0; \\ \gamma + 2S_{12} - S_{22}^2 / \mu = 0; \\ S_{11} - S_{12}S_{22} / \mu = 0. \end{cases} \quad (8)$$

Then the elements of matrix S taking into account its positive certainty are:

$$S_{12} = \sqrt{\alpha\mu}; \quad S_{22} = \sqrt{\mu(\gamma + 2S_{12})}; \quad S_{11} = S_{12}S_{22} / \mu. \quad (9)$$

Synthesizing function is determined by matrix expression

$$U = -G^{-1}B'SX = -\left[\frac{1}{\mu}\right] \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\mu}(S_{12}x_1 + S_{22}x_2).$$

After substituting elements S_{12}, S_{22} , synthesizing function has the form

$$U = ((\gamma + 2\sqrt{\alpha\mu}) / \mu)^{1/2} \dot{Y} + (\alpha / \mu)^{1/2} Y, \quad (10)$$

If we substitute synthesizing function (10) into equation (3), we will obtain

$$\ddot{Y} + a_1 \dot{Y} + a_2 Y = 0, \quad (11)$$

where $a_1 = ((\gamma + 2\sqrt{\alpha\mu}) / \mu)^{1/2}$; $a_2 = (\alpha / \mu)^{1/2}$.

We will write characteristic equation down for (11)

$$r^2 + a_1 r + a_2 = 0, \quad (12)$$

Characteristic equation of the differential equation of the model-analogue (2) has the form

$$r^2 + a_1 r + a_2 = 0, \quad (13)$$

where $a_1 = \beta / m$; $a_2 = c / m$.

From comparison of a_1, a_2 in equations (12) and (13), we obtain analytical dependences for determining of design parameters – coefficients of BPA growth and retirement;

$$c = m(\alpha / \mu)^{1/2}; \quad \beta = m((\gamma + 2\sqrt{\alpha\mu}) / \mu)^{1/2}. \quad (14)$$

As it is seen, design parameters are determined through weight coefficients of the quadratic functional of quality.

2. Substantiation of selection of weight factors of quadratic functional of quality [9].

Selection of weight factors of quadratic functional of quality is a complex problem, the elements of matrices P and G are often selected by trial and error method, which significantly impedes the synthesis of optimal systems by this criterion.

In economic systems, aperiodic law of development is close to optimal. Synthesizing function must be physically realizable. To verify this, it is necessary to have dependences between weight factors of the quadratic functional of quality and the range of their

changes. Dependences are established from formulas (14) in view of the fact that coefficient of growth of BPA is normally by two orders of magnitude lower than the coefficient of retirement.

$$\alpha = \mu c^2 / m^2; \quad \gamma = 10^4 \alpha - 2\sqrt{\alpha\mu}. \quad (15)$$

Let us associate weight factors of functional of quality with parameters of the process of gross product manufacturing. The range of a change of parameters of the model of the enterprise's lifecycle, providing for sustainable operations of an enterprise is: $\beta = 0,08 \div 0,3$; $m = 0,2 \div 4,9$; $s = 0,002 \div 0,052$ [3]. In this case, values of weight factors of the functional of quality change in the range:

$$\mu = 10000 \div 20000; \quad \alpha = 0,005 \div 0,045; \quad \gamma = 10 \div 70.$$

Requirement for positive certainty of matrix S allows establishing of an additional dependence between weight factors. Let us have $\alpha = 0,005$; $\gamma = 10$; $\mu = 10000$.

Then

$$S_{12} = \sqrt{\alpha\mu} = 7,071; \quad S_{22} = \sqrt{\mu(\gamma + 2S_{12})} = 491,347;$$

$$S_{11} = S_{12}S_{22} / \mu = 0,347.$$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}; \quad \Delta_1 = |S_{11}| = 0,347 > 0;$$

$$\Delta_2 = \begin{vmatrix} 0,347 & 7,071 \\ 7,071 & 491,347 \end{vmatrix} = 120,498 > 0.$$

All minors are more than zero, therefore, matrix S is positively determined. For positive certainty of the matrix, it is necessary to satisfy conditions $S_{11}S_{22} > S_{12}^2$ or $S_{22}^2 > \mu S_{12}$, which is possible at the following dependence between weight factors $\gamma > -\sqrt{\alpha\mu}$.

Table 1 shows calculation of optimal values of coefficient of BPA growth from formulas (14), Table 2 shows the values of coefficient of BPA retirement. Original data include: capital ratio $m=1,5$; $\mu = 10000$; 15000; 20000; $\gamma = 10$; 30; 50; 70.

Table 1

Region of optimal values of coefficient of BPA growth

	A	B	C	D
28	m	1,5		
29	mu	10000	15000	20000
30	alfa	c		
31	0,005	0,00106	0,00087	0,00075
32	0,01	0,0015	0,00122	0,00108066
33	0,015	0,00184	0,0015	0,00129904
34	0,02	0,00212	0,00173	0,0015
35	0,025	0,00237	0,00194	0,00167705
36	0,03	0,0026	0,00212	0,00183712
37	0,035	0,00281	0,00229	0,00198431
38	0,04	0,003	0,00245	0,00212132
39	0,045	0,00318	0,0026	0,00225

Table 2
Region of optimal values of coefficient of BPA retirement

	A	B	C	D	E
41	m	1,5			
42	mu	10000			
43	gamma	10	30	50	70
44	alfa	beta			
45	0,005	0,0737	0,09966	0,12013318	0,13759
46	0,01	0,08216	0,10607	0,125499	0,1423
47	0,015	0,0881	0,11073	0,12946564	0,14581
48	0,02	0,09281	0,11452	0,1327176	0,14871
49	0,025	0,09677	0,11775	0,13551799	0,15121
50	0,03	0,10022	0,1206	0,13800083	0,15344
51	0,035	0,10329	0,12316	0,14024525	0,15546
52	0,04	0,10607	0,1255	0,14230249	0,15732
53	0,045	0,10861	0,12766	0,14420798	0,15905

The data of Table 2 indicate that the values of the coefficient of BPA retirement are less than 0.3. The value of the coefficient of BPA growth (Table 1) is by two orders of magnitude less than the values of the coefficient of retirement, i.e., optimal parameters provide for physical implementation of the process of gross product output.

Fig. 1–2 show dependences between design parameters and weight factors of quality functional [3]

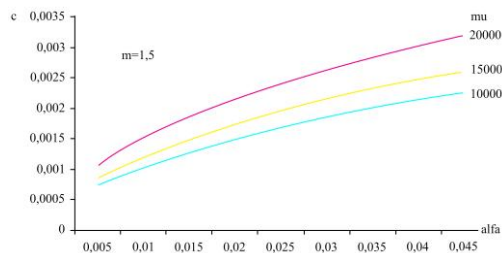


Fig. 1. Dependence of the coefficient of BPA growth on weight coefficients of quality functional

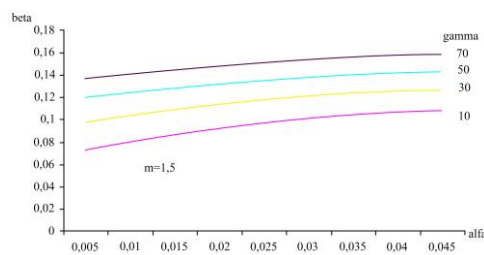


Fig. 2. Dependence of the coefficient of BPA retirement on weight coefficients of quality functional

3. Algorithm of optimal filters of Kalman-Bucy

The state of the studied process is described by equations:

$$\begin{cases} \frac{dX(t)}{dt} = F(t)X(t) + L(t)N(t), \\ Y(t) = H(t)X(t) + V(t). \end{cases} \quad (16)$$

Kalman proved that the solution of the problem of optimal filtration is assessment $\hat{X}(t)$ – output vector of the dynamic process, which is described by differential equation

$$\frac{d\hat{X}}{dt} = F(t)\hat{X}(t) + C(t)[Y(t) - H(t)\hat{X}(t)], \quad \hat{X}(t_0) = \bar{X}_0. \quad (17)$$

Expression (17) is called a differential equation of the filter, which is «disturbed» by observed signal $Y(t)$ and at the output gives the best linear assessment $\hat{X}(t)$ of vector of state $X(t)$ of the dynamic process. Matrix $C(t)$ is called matrix coefficient of strengthening of the optimal filter and is determined by expression $C(t) = K(t)H'(t)R^{-1}(t)$, where R^{-1} is the reverse matrix in relation to matrix $R(t)$; $K(t)$ is the correlation matrix of the error of optimal assessment [1]. Matrix $K(t)$ is symmetric by size $n \times n$, its each element is represented in the form of $K_{jl}(t) = M\{[x_j(t) - \bar{x}_j][x_l(t) - \bar{x}_l]\}$, $(j, l = 1, 2, \dots, n)$. R. Kalman obtained the nonlinear differential equation to determine matrix $K(t)$

$$\frac{dK}{dt} = F(t)K(t) + K(t)F'(t) - \quad (18)$$

$$K(t)H'(t)R^{-1}(t)H(t)K(t) + L(t)Q(t)L'(t)$$

which is called a correlation or a dispersion equation. It is system $n(n+1)/2$ of differential equations of the Riccati type. Thus, solution of the optimal filtration problem is derived from the system of equations (18), (17), (16), which form the Kalman algorithm. In this case, original conditions must be assigned in the form of initial assessment $X(t_0) = \bar{X}_0$ of the state of the process and original value of correlation matrix $K(t_0)$. Matrix of parameters of the optimal filter is formed from equation (17)

$$F_{opt} = F(t) - C(t)H(t) = F - KH'R^{-1}H. \quad (19)$$

When implementing the Kalman filter, it is important to provide for its stability. R. Kalman and R. Bucy showed that the optimal filter is a uniformly asymptotically stable dynamic process. In this case, solution of a correlation equation represents a stable computation process, not sensitive to insignificant errors.

Research results

1. The process of gross product output of an enterprise as a control object

To prove that the process of gross product manufacturing by an enterprise is a control object, the concepts of controllability and observability of dynamic systems are used. In paper [7], the methods of obtaining controllability and observability criteria of linear dynamic systems are clearly presented.

Research into observability of the process of gross product manufacturing was conducted on the mathematical model

$$\ddot{Y} + a_1\dot{Y} + a_2Y + u(t) = 0.$$

Record of the original process in the matrix form

$$\dot{X} = AX + BU; Z = HX,$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}; B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; U = [u(t)].$$

Record of the conjugate process

$$\begin{aligned} \dot{X}^* &= -A'X^* - H'U^*; \\ Z^* &= B'X^*, \end{aligned}$$

where

$$A' = \begin{bmatrix} 0 & -a_2 \\ 1 & -a_1 \end{bmatrix}; B' = [0 \quad -1]; H' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; X^* = \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix}; U^* = [u^*(t)].$$

Controllability matrix of the conjugate process is referred to as the matrix of observability of the original process, i.

$$e. Q_n = Q_y^*.$$

$$\text{Rank } Q_y^* = \text{Rank} \begin{bmatrix} H' & A'H' \\ 0 & 0 \end{bmatrix} = \text{Rank} \begin{bmatrix} 1 & 0 & 0 & -a_2 \\ 0 & 1 & 1 & -a_1 \end{bmatrix} = 2.$$

The highest order of the minor of this matrix that is different from zero is equal to the size of vector X^* . Therefore, the conjugate process is fully controllable, and the original process is fully observable, i. e. the studied process of gross product manufacturing by an enterprise is a control object [5].

2. Determining the parameters of the process of gross product output of an enterprise

Analytical dependences for determining of design parameters – coefficients of growth and retirement BPA:

$$c = m(\alpha / \mu)^{1/2}; \quad \beta = m((\gamma + 2\sqrt{\alpha\mu}) / \mu)^{1/2}.$$

Dependences between weight factors of quadratic functional of quality and the range of their change:

$$\alpha = \mu c^2 / m^2; \quad \gamma = 10^4 \alpha - 2\sqrt{\alpha\mu}; \quad \gamma > -\sqrt{\alpha\mu}.$$

The range of the change in parameters of the model of the life cycle of an enterprise, providing for sustainable functioning of an enterprise [3]:

$$\beta = 0,08 \div 0,3; \quad m = 0,2 \div 4,9; \quad s = 0,002 \div 0,052.$$

In this case, values of weight factors of the functional of quality change in the range:

$$\mu = 10000 \div 20000; \quad \alpha = 0,005 \div 0,045; \quad \gamma = 10 \div 70.$$

Selection of design parameters from the region of optimal values is performed by analysis of the steady law of the process development [10].

Let us verify the stability of the process of gross product manufacturing by the roots of the characteristic equation. The process is steady under the aperiodic law of development. In this case, the roots of the characteristic equation must be negative and different. We select $\beta = 0,1086$ from Table 2, $c = 0,00106$ at $m = 1,5$ from Table 1, then the roots of equation (13) $r_1 = -0,0117$; $r_2 = -0,0607 = 0,1086$, from table 1 =

0.00106 if $\beta = 1,5$, then the roots of the equation (13) = 0.0117; = 0.0607.

Therefore, the process of gross product manufacturing of the enterprise is steady.

3. Obtaining of the optimal filter of the process of gross product manufacturing

We will determine the matrix of the parameters of the optimal filter for gross product manufacturing by an enterprise

$$\begin{aligned} m\ddot{Y} + \beta\dot{Y} + cY &= N(t), \\ \ddot{Y} + a_1\dot{Y} + a_2Y &= n(t), \end{aligned} \quad (20)$$

where

$a_1 = \beta/m$; $a_2 = c/m$; $n(t) = N(t)/m$; $N(t)$ are the volume of external investments of an enterprise.

Let us present equation (20) in the Cauchy form, accepting $x_1 = Y$, $x_2 = \dot{Y}$

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_2x_1 - a_1x_2 + n(t). \end{cases} \quad (21)$$

Let us assume that during the process of gross product manufacturing, motions and velocities are measured, that is, all components of the vector of the process state are “observed”. In this case, the dynamic process will be presented by differential equations

$$\frac{dX(t)}{dt} = FX(t) + LN(t); \quad (22)$$

$$Z(t) = HX(t) + V(t),$$

where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is the vector-column of the process

state; $F = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}$ is the matrix of the process

parameters; $N = \begin{bmatrix} 0 \\ n(t) \end{bmatrix}$ is the vector-column of

external investments; $L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity

matrix at vector $N(t)$; $H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the identity

matrix characterizes relations of the observed vector with state vector; $V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is the vector-column of measurement errors.

To determine matrix $K(t)$ of the matrix coefficient of strengthening of the optimal filter, we will use the Riccati equation

$$\frac{dK}{dt} = FK(t) + K(t)F' - K(t)H'R^{-1}HK(t) + LQL', \quad (23)$$

where

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}; Q = \begin{bmatrix} 0 & 0 \\ 0 & q \end{bmatrix}; R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix};$$

$$R^{-1} = \begin{bmatrix} 1/r_1 & 0 \\ 0 & 1/r_2 \end{bmatrix}.$$

Substituting matrices in equation (23) and performing matrix operations, we will obtain the system of the Riccati equations

$$\dot{K}_{11} = 2K_{12} - K_{11}^2/r_1 - K_{12}^2/r_2;$$

$$\dot{K}_{12} = K_{22} - a_2K_{11} - a_1K_{12} - K_{11}K_{12}/r_1 - K_{12}K_{22}/r_2$$

$$\dot{K}_{22} = -2(a_2K_{12} + a_1K_{22}) - K_{12}^2/r_1 - K_{22}^2/r_2 + q$$

The expression for the filter misalignment factor is

$$C(t) = K(t)H'R^{-1} = \begin{bmatrix} K_{11}/r_1 & K_{12}/r_2 \\ K_{21}/r_1 & K_{22}/r_2 \end{bmatrix}. \quad (24)$$

Then the matrix of parameters of the optimal filter will take the form

$$F_{opt} = F - C(t)H = \begin{bmatrix} -K_{11}/r_1 & 1 - K_{12}/r_2 \\ -a_2 - K_{12}/r_1 & -a_1 - K_{22}/r_2 \end{bmatrix}. \quad (25)$$

Analyzing the matrix of parameters of the optimal filter, we conclude that in this case coefficients of BPA growth and retirement must change [5]. If the values of the correlation functions K_{11} , K_{12} , K_{22} approximate or are equal to zero, the original process will be close to optimal.

We will write down the matrix of parameters of the optimal filter as

$$F_{opt} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}.$$

Characteristic equation of the optimal filter

$$D = \begin{vmatrix} A_1 - \lambda & A_2 \\ A_3 & A_4 - \lambda \end{vmatrix} = 0 \quad \text{or}$$

$$\lambda^2 - (A_1 + A_4)\lambda + A_1A_4 - A_2A_3 = 0.$$

Therefore, coefficients of growth and retirement of BPA can be determined from formulas

$$c = m(A_1A_4 - A_2A_3);$$

$$b = -m(A_1 + A_4). \quad (26)$$

To obtain numerical values for parameters, it is necessary to solve the system of Riccati equations on a computer and substitute values K_{11} , K_{12} , K_{22} , corresponding to the established process, in (25).

Analysis [7] of the matrix of the optimal filter parameters (25) shows that the values of coefficients of growth and retirement of BPA must change over time, i.e. coefficients of retirement and growth of BPA should not be constant at the stages of the lifecycle and an enterprise must develop intensively.

Conclusions

Thus, as a result of the conducted research:

1. Based on the concepts of observability and controllability, it was proved that the process of gross product manufacturing by an enterprise is a control object.
2. Analytical dependences for coefficients of growth and retirement of BPA were determined using the matrix method of dynamic programming[6]
3. Selection of weight coefficients of quadratic quality functional was performed. The selection was based on the requirement to get aperiodic law of development of the process of gross product output and the physically implemented synthesizing function [11].
4. The regions of optimal values of design parameters were determined (Table 1 and Table 2). The data of Table 2 indicate that the value of the coefficient of BPA retirement is less than 0.3. The values of the coefficient of BPA growth (Table 1) are by two orders of magnitude less than the values of the coefficient of retirement, i.e. the optimal values of parameters provide for physical feasibility of the process of gross product output.
5. Selection of design parameters from the region of optimal values, which is provided by the steady law of process development, was substantiated. The growth coefficient is 0.00106 and BPA retirement coefficient is 0.1086 at fund capacity of 1.5.
6. Optimal filter of Kalman-Bucy for the process of gross product manufacturing by an enterprise was constructed. The values of coefficients of BPA growth and retirement of the filter matrix can change over time, i.e., these coefficients should not be constant at the stages of the lifecycle and an enterprise must develop intensively.
7. Modeling of the process of gross product output with optimal values of coefficients of BPA growth and retirement was performed in modeling system MVTU 3.7. The diagram of the process of gross product output without external investments (Fig. 5) and transitional characteristic (Fig. 7) prove the sustainability of the process.

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